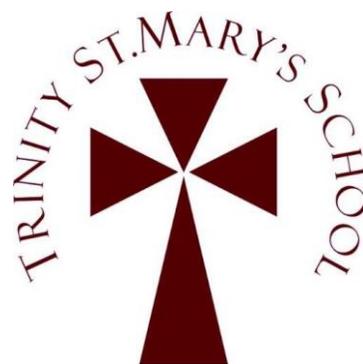


Trinity St. Mary's Church of England
Voluntary Aided Primary School

Calculation Policy



Summer 2022

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum.

This calculation policy should be used to support children to develop a deep understanding of number and calculation. It has been designed to teach children through the use of concrete, pictorial and abstract representations.

Maths Mastery and our 'Dive Deeper' approach

At the centre of the mastery approach to the teaching of maths is the belief that all children have the potential to succeed and the aim is to develop children's learning at the same pace. As much as possible, children should be accessing the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Differentiation should primarily be through support, scaffolding and deepening, not through task. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials, pictorial representations and varied approaches. Children of all ages and at all stages of learning apply Dive Deeper questions to their maths in order to broaden their thinking, deepen their understanding and increase their ability to make links between mathematical concepts. Initially with support and guidance, and then independently, children pick a question such as: represent it using manipulatives, solve it using a pictorial method, write the question as a word problem, explain it to an alien, prove your answer using a different method, what mistake could be made when answering this question, make up a similar question for a friend, what is the number sentence for this question or write a maths story for the problem, and apply it to one of the fluency questions they have been working on. This approach allows children to sit with their learning longer and devote more thinking time to understanding the structure of the concept, as well as unpicking any of their own misconceptions. Dedicated problem-solving lessons are also a central part of teaching. These allow students to wonder why things are, to inquire, to search for solutions, to resolve incongruities and develop resilience. This develops pupils' understanding of why something works so that they truly have an appreciation of what they are doing rather than just learning to repeat routines without grasping what is happening.

Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). In certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant, real objects, apparatus, pictures of diagrams) and explained carefully. High and consistent expectations of the mathematical language used are essential, with teachers only accepting what is correct.

'The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.' - 2014 Mathematics Programme of Study

This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

Using the Concrete-Pictorial-Abstract Approach:

Children develop an understanding of a mathematical concept through the three steps of: concrete, pictorial and abstract (CPA) approach. Reinforcement is achieved by going back and forth between these representations. The CPA should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils. New units should be introduced to the whole class using manipulatives wherever possible in every year group.

Concrete Representation:

This is the first step in a child's learning. The child is introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial Representation:

Once the child has sufficiently understood the 'hands on' experience, they can be progressed onto relating them to pictorial representations, such as a diagram or a picture of the problem.

Abstract Representation:

This is the third step in a child's learning. The child should now be capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

The Counting Principles (Gelman & Gallistel 1978)

The One-One Principle

This involves children assigning one number name to each object that is being counted. Children need to ensure that they count each object only once ensuring that they have counted every object.

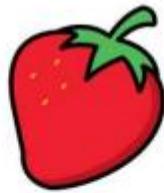
Children will sometimes count objects more than once or miss an object out that needs to be counted. Encourage children to line up objects and touch each one as they count saying one number name per object. This will also help to avoid children counting more quickly than they touch the objects which again shows they have not grasped one-one correspondence.



1



2



3



4



5

The Stable Order Principle

Children understand when counting, the numbers have to be said in a certain order.

Children need to know all the number names for the amount in the group they are counting. Teachers can therefore encourage children to count aloud to larger numbers without expecting them to count that number of objects immediately.

The Cardinal Principle

Children understand that the number name assigned to the final object in the group is the total number of objects in that group.

In order to grasp this principle, children need to understand the one-one and stable-order principle. From a larger group, children select a given number and count them out. When asked 'How many?', children should be able to recall the final number they said. Children who have not grasped this principle will recount the whole group again.

The Abstraction Principle

This involves children understanding that anything can be counted including things that can not be touched including sounds and movements e.g. jumps.

When starting to count, many children rely on touching objects in order to count accurately. Teachers can encourage abstraction on a daily basis by counting claps or clicks. They can also count imaginary objects in their heads to encourage counting on, this involves the children visualising objects.

The Order-Irrelevance Principle

This involves children understanding that the order we count a group of objects in is irrelevant. There will still be the same number.

Encourage children to count objects, left to right, right to left, top to bottom and bottom to top. Once children have counted a group, move the objects and ask children how many there are, if they count them all again they have not fully grasped this principle.

Progression in Calculations - Addition

Nursery / Pre-School

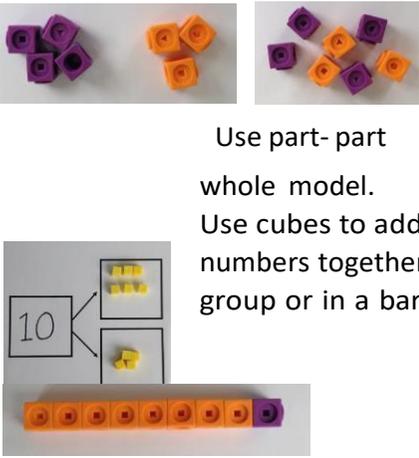
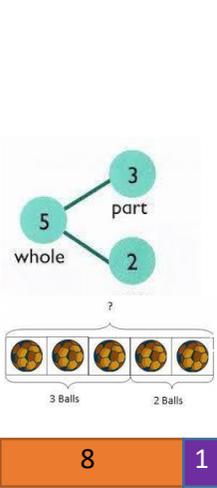
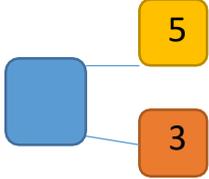
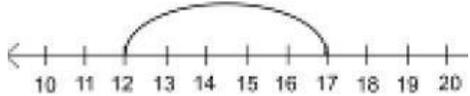
Before addition can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting, number order and number recognition through practical activities and games. This is taught through child-initiated games such as hide and seek and I spy. Children also learn how to count 1-1 (pointing to each object as they count) and that anything can be counted, for example, claps, steps and jumps. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc.

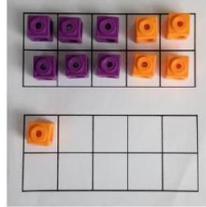
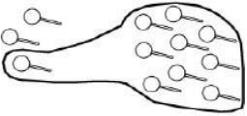
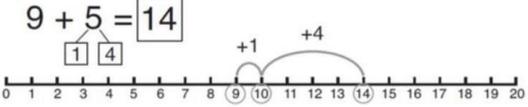
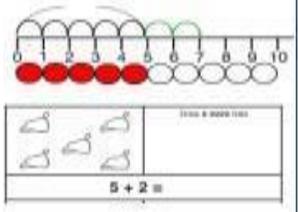
Reception / EYFS

Before addition can be introduced, children in Reception build on concepts taught in Nursery by working through the number objectives in the 40 – 60 month band of Development Matters. Children need to have a secure knowledge of number in order to begin addition. Children are then introduced to the concept of addition through practical games and activities. Children act out addition sums to physically add two groups of objects together and use arm gestures to represent the signs + and =. This is reinforced by opportunities provided in the outdoor area for the children to use addition e.g. adding together groups of building blocks, twigs etc. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definition. An example of this is "addition means we add two groups together / we put 2 lots of objects together. Equals means we find out how many we have got altogether. 3 add 2 equals 5! We have got 5 altogether". Adults support children in recording their addition sums in the written form on whiteboards and in their maths books.

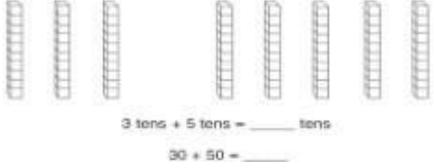
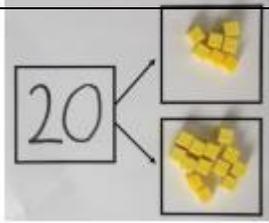
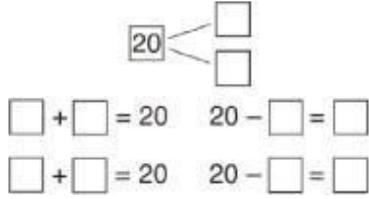
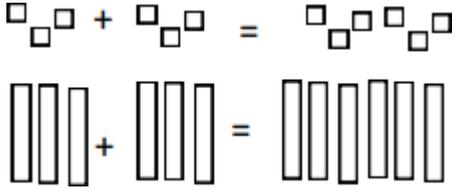
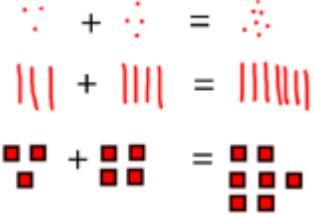


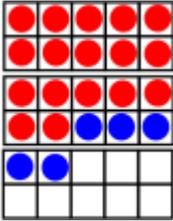
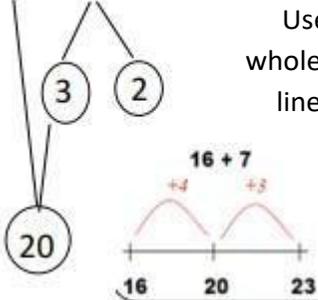
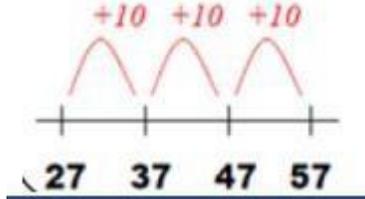
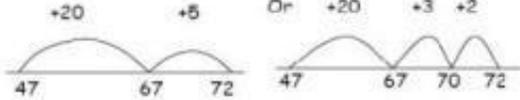
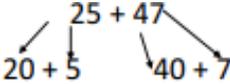
EYFS - Year 1 – Addition

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model.</p>	 <p>Use part-part whole model. Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$ $10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the bigger number and counting on.</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$12 + 5 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>

<p>Regrouping to make 10.</p>	 <p>$6 + 5 = 11$</p>  <p>Start with the bigger number and use the smaller number to make 10.</p>	 <p>Use pictures or a number line.</p> <p>Regroup or partition the smaller number to make 10.</p> <p>$3 + 9 =$</p> <p>$9 + 5 = 14$</p> 	<p>$7 + 4 = 11$</p> <p>“If I am at seven, how many more do I need to make 10? How many more do I add on now?”</p>
<p>Represent & use number bonds and related subtraction facts within 20.</p>	 <p>2 more than 5.</p>		<p>Emphasis should be on the language:</p> <p>“1 more than 5 is equal to 6”</p> <p>“2 more than 5 is 7”</p> <p>“8 is 3 more than 5”</p>

Year 2 – Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten.	<p style="text-align: center;">$50 = 30 + 20$</p>  <p style="text-align: center;">Model using dienes and bead strings.</p>	 <p style="text-align: center;">Use representations for base ten.</p>	<p style="text-align: center;">$20 + 30 = 50$</p> <p style="text-align: center;">$70 = 50 + 20$</p> <p style="text-align: center;">$40 + \underline{\quad} = 60$</p>
Use known number facts including different combinations of tens & ones of any 2 digit number. (Part part whole)	 <p style="text-align: center;">Children explore ways of making numbers.</p>		<p style="text-align: center;">Include teaching of the inverse of addition and subtraction:</p> <p style="text-align: center;">$\square + 1 = 16$ $16 - 1 = \square$</p> <p style="text-align: center;">$1 + \square = 16$ $16 - \square = 1$</p>
Use known facts.		 <p style="text-align: center;">Children draw representations of H, T & O.</p>	<p style="text-align: center;">$3 + 4 = 7$</p> <p style="text-align: center;"><i>Leads to</i></p> <p style="text-align: center;">$30 + 40 = 70$</p> <p style="text-align: center;"><i>Leads to</i></p> <p style="text-align: center;">$300 + 400 = 700$</p>

Use bar models.	 $3 + 4 = 7$	 $7 + 3 = 10$	<table border="1" data-bbox="1671 220 2040 328"> <tr> <td>23</td> <td>25</td> </tr> <tr> <td colspan="2">?</td> </tr> </table> $23 + 25 = 48$	23	25	?	
23	25						
?							
Add a two digit number and ones.	 <p>Use ten frame to make 'magic ten'.</p> <p>Children explore the patterns:</p> $17 + 5 = 22$ $27 + 5 = 32$	$17 + 5 = 22$ <p>Use part part whole and number line to model.</p> 	$17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ <table border="1" data-bbox="1839 563 2078 651"> <tr> <td colspan="2">22</td> </tr> <tr> <td>17</td> <td>5</td> </tr> </table>	22		17	5
22							
17	5						
Add 2 digit numbers and tens.	 $25 + 10 = 35$ <p>Explore that the ones digit does not change.</p>	$27 + 30$ 	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \underline{\quad} = 57$				
Add two 2-digit numbers.	 <p>Model using dienes, place value counters and numicon.</p>	 <p>Use number line and bridge ten using part whole if necessary.</p>	$25 + 47$  $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$				

Add three 1-digit numbers.

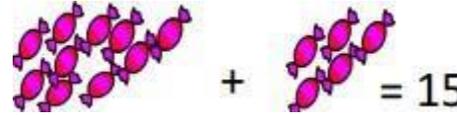
$4 + 7 + 6 = 17$
Put 4 and 6 together to make 10.
Add on 7.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



Regroup and draw representation.



$$\begin{array}{r} \textcircled{4} + 7 + \textcircled{6} = \boxed{10} + \boxed{7} \\ 10 \\ = \boxed{17} \end{array}$$

Combine the two numbers that make/bridge ten, then add on the third.

Rapid Recall

(addition and subtraction)

- Bonds within 10
- Bonds within 20
- Bonds to 100 (multiples of 10)
- Add single-digit to make a multiple of 10

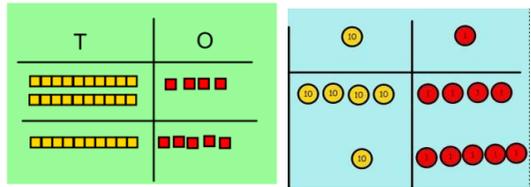
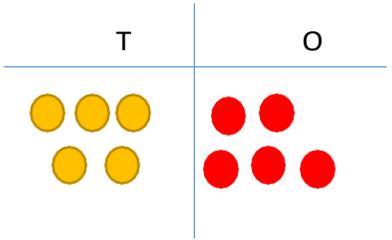
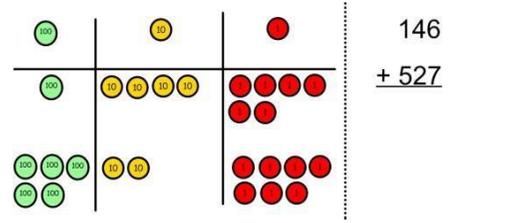
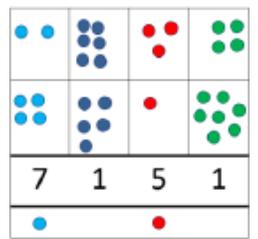
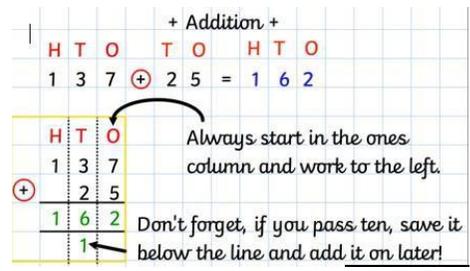
Strategies

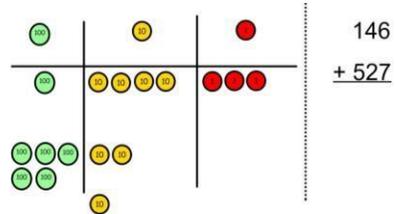
Add/subtract 9, 19, 29...

Partitioning

- Add near doubles
- Reorder
- Count on/back in 10s

Year 3 – Addition

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Addition – no regrouping (friendly numbers)</p> <p>Add 2 or 3 digit numbers.</p>	<p style="text-align: center;">$24 + 15 =$</p>  <p>Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p>	<p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p> 	<p>Add the ones first, then the tens, then the hundreds:</p> $\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Children use the 'steps to success' to format their calculation:</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center; color: red;">*Steps for Success*</p> <ol style="list-style-type: none"> <li style="color: red;">1. Write your calculation, label your digits and circle the operation. <li style="color: yellow;">2. Check your operation, choose your method and set it up below. Remember to leave plenty of room for working out! <li style="color: green;">3. Use the method to calculate the answer. <li style="color: blue;">4. Write the answer at the end of the calculation. </div>
<p>Column Addition – with regrouping.</p>	<p>Make both numbers on a place value grid.</p>  <p style="text-align: right;">$146 + 527$</p> <p>Add up the units and exchange 10 ones for one 10.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p> 	<p>Children follow the 'Steps to Success' to regroup and form the calculation correctly:</p> 



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

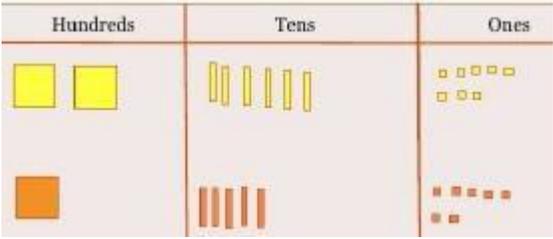
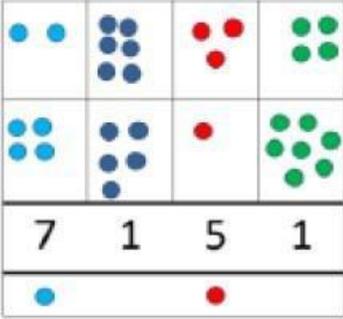
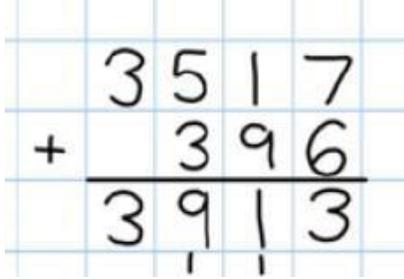
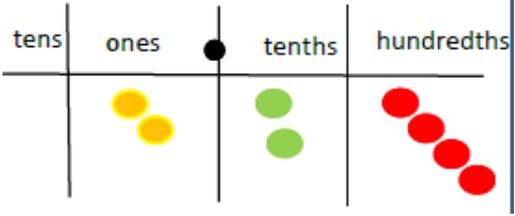
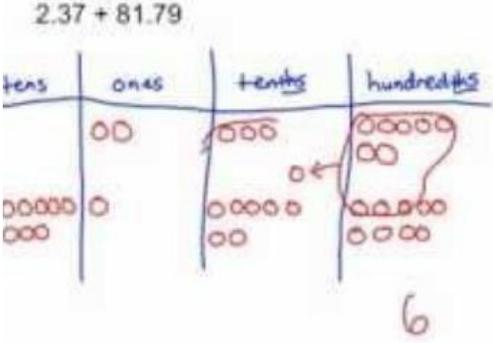
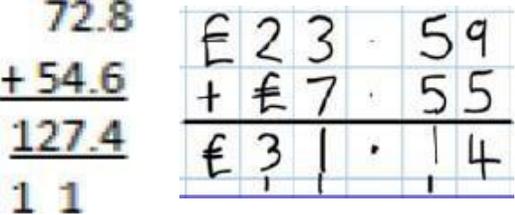
As the children move on, introduce decimals with the same number of decimal places and different. Money is used for context.

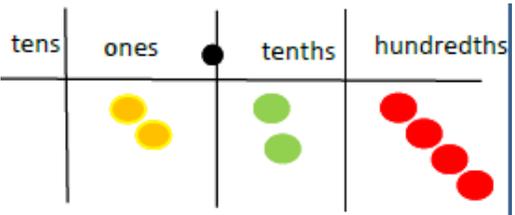
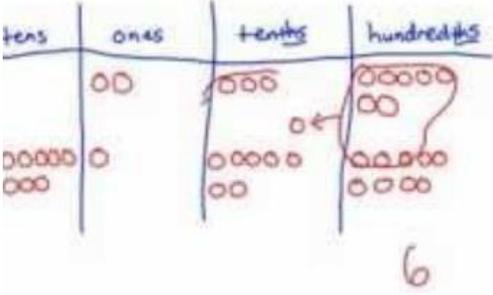
$$\begin{array}{r}
 72.8 \\
 + 54.6 \\
 \hline
 127.4
 \end{array}$$

$$\begin{array}{r}
 \pounds 23.59 \\
 + \pounds 7.55 \\
 \hline
 \pounds 31.14
 \end{array}$$

$$\begin{array}{r}
 23.361 \\
 9.080 \\
 + 1.300 \\
 \hline
 33.741 \\
 \hline
 21.2
 \end{array}$$

Years 4-6 – Addition

Objective & Strategy	Concrete	Pictorial	Abstract
<p><u>Year 4</u> Add numbers with up to 4 digits</p>	<p>Children continue to use dienes or place value counters to add, exchanging ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand.</p> 	<p>Draw representations using place value grid.</p> 	<p>Continue from previous work to carry hundreds as well as tens.</p> <p>Relate to money and measures.</p> 
<p><u>Year 5</u> Add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>(As year 4)</p> <p>Introduce decimal place value counters and model exchange for addition.</p> 	<p>(As year 4)</p> <p>$2.37 + 81.79$</p> 	<p>(As year 4)</p> 

<p><u>Year</u> <u>6</u></p> <p>Add several numbers of increasing complexity.</p> <p>Include adding money, measure and decimals with different numbers of decimal points.</p>	<p>(As year 5)</p> <p>Introduce decimal place value counters and model exchange for addition.</p> 	<p>(As year 5)</p> <p>$2.37 + 81.79$</p> 	<p>Insert zeros for place holders.</p> <table border="1" data-bbox="1590 271 2139 502"> <tr> <td>81,059</td> <td>23.361</td> </tr> <tr> <td>3,668</td> <td>9.080</td> </tr> <tr> <td>15,301</td> <td>59.770</td> </tr> <tr> <td>+ 20,551</td> <td>+ 1.300</td> </tr> <tr> <td><hr/></td> <td><hr/></td> </tr> <tr> <td>120,579</td> <td>93.511</td> </tr> <tr> <td>1 1 1 1</td> <td>2 1 2</td> </tr> </table>	81,059	23.361	3,668	9.080	15,301	59.770	+ 20,551	+ 1.300	<hr/>	<hr/>	120,579	93.511	1 1 1 1	2 1 2
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<p><u>Year</u> <u>4</u></p> <p>Rapid Recall (addition and subtraction)</p>	<div style="display: flex; justify-content: space-between;"> <ul style="list-style-type: none"> • Sums/differences – multiples of 10/100/1000 • Doubles – within 100 • Add/subtract multiples of 10/100/1000 <div style="border: 1px solid black; border-radius: 50%; padding: 20px; width: 30%; text-align: center;"> <p><u>Strategies</u></p> <p>Partition</p> <p>Small difference</p> <ul style="list-style-type: none"> • Bridging • Round & adjust </div> </div>																

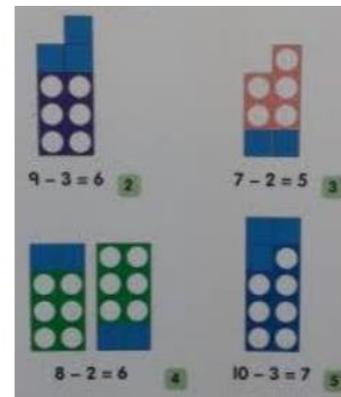
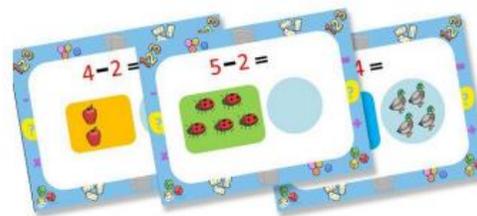
Progression in Calculations – Subtraction

Nursery / Pre-School

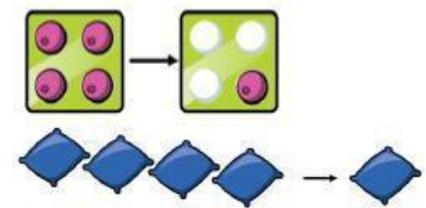
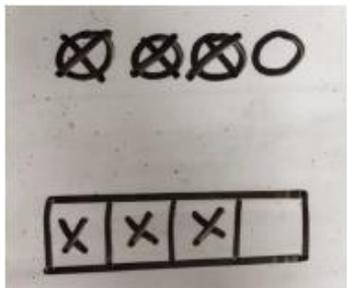
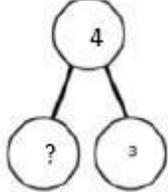
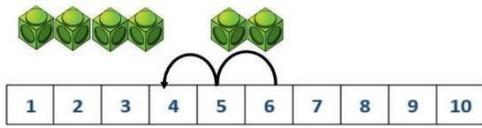
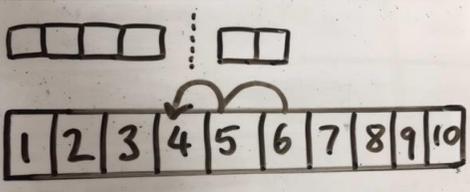
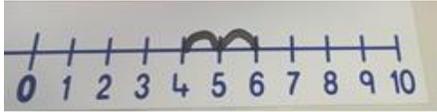
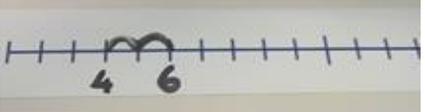
Before subtraction can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting backwards. This is taught through child-initiated games indoors and outdoors such as acting out counting songs and running races (children shouting “5,4,3,2,1,0 - GO!”).

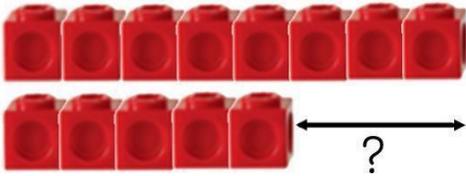
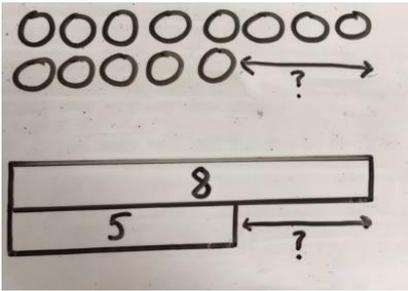
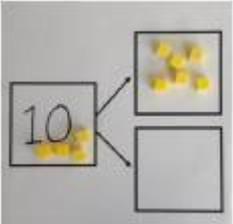
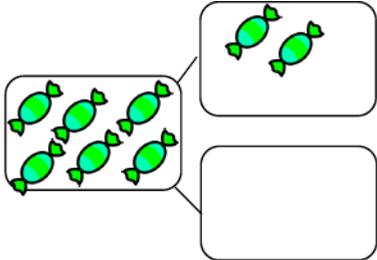
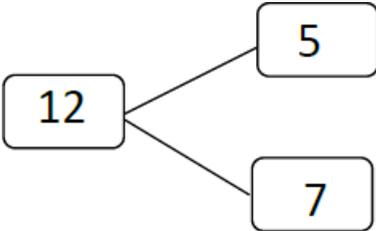
Reception / EYFS

Before subtraction can be introduced, children in Reception build on concepts taught in Nursery by working through the number objectives in the 40 – 60 month band of Development Matters. Children need to have a secure knowledge of number in order to begin subtraction. Children are then introduced to the concept of subtraction through practical games and activities. Children act out subtractions to physically subtract a number of objects from a group. Children use arm gestures to represent the signs - and =. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. Children build on their previous knowledge of ‘less’ by learning that subtracting means taking away a certain number of objects from a group (leaving them with less objects). Adults model subtraction vocabulary supported by age appropriate definition. An example of this is “subtraction means we take away objects from a group / we have 11 got less objects now. Equals means we find out how many we have got left. Wow! We have only got 3 left!” Adults support children in recording their subtractions in the written form on whiteboards and in their maths books.

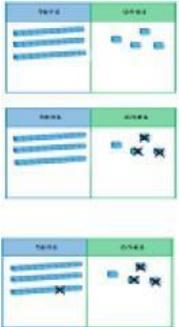
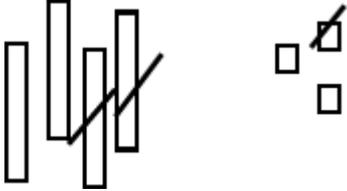
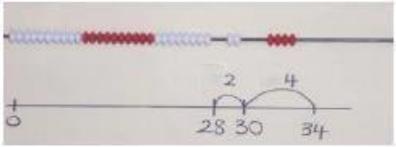
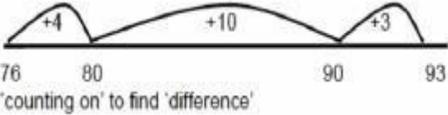


EYFS - Year 1 – Subtraction

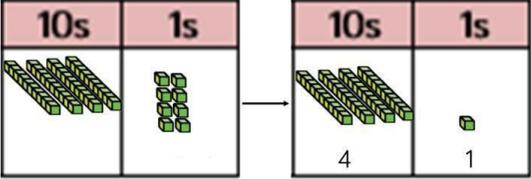
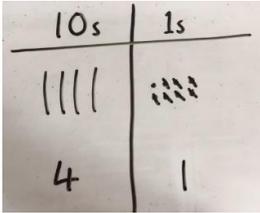
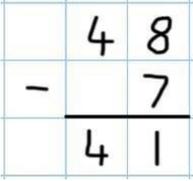
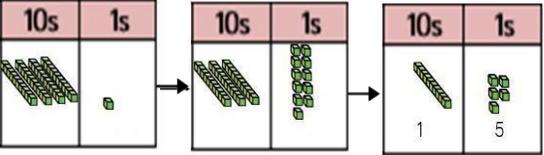
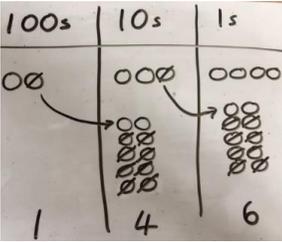
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Taking away ones from a whole.</p>	<p>Use physical objects, counters, cubes etc. to show how objects can be taken away.</p> <p>$4 - 3 = 1$</p> 	<p>Cross out drawn objects to show how many has been taken away. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> <p> $= 4 - 3$</p>  
<p>Counting back.</p>	<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.</p>  

<p>Finding the difference.</p>	<p>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).</p> <p>Calculate the difference between 8 and 5:</p> 	<p>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.</p> 	<p>Find the difference between 8 and 5. $8 - 5$, the difference is...</p> <p>Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.</p>
<p>Represent and use number bonds and related subtraction facts within 20.</p> <p>(Part part whole model)</p>	<p>Link to addition – use the PPW model to model the inverse.</p>  <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	<p>Use pictorial representations to show the parts.</p> 	<p>Move to using numbers within the part whole model.</p> 

Year 2 – Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Partitioning to subtract – without regrouping.</p> <p>(friendly numbers)</p>	<p>Use dienes to show how to partition the number when subtracting without regrouping.</p> <p>$34 - 13 = 21$</p> 	<p>Children draw representations of dienes and cross off.</p> <p>$43 - 21 = 22$</p> 	<p>$43 - 21 = 22$</p>
<p>Making ten.</p> <p>(crossing one ten, crossing more than oneten, crossing the hundreds)</p>	<p>Use a bead string to model counting to the next ten and the rest.</p> <p>$34 - 28 =$</p> 	<p>Use a number line to count on to the next ten and then the rest.</p> 	<p>$93 - 76 = 17$</p>

Year 3 – Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract																	
<p>Column subtraction without regrouping.</p> <p>(friendly numbers)</p>	<p>Column method using base ten.</p> 	<p>Children to represent the base 10 pictorially.</p> 	<p>Column method or children could count back 7.</p>  <p>Children use their 'Steps to Success' to format the question correctly:</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p style="text-align: center;">*Steps for Success*</p> <ol style="list-style-type: none"> 1. Write your calculation, label your digits and circle the operation. 2. Check your operation, choose your method and set it up below. Remember to leave plenty of room for working out! 3. Use the method to calculate the answer. 4. Write the answer at the end of the calculation. </div>																	
<p>Column subtraction with regrouping.</p>	<p>Column method using base 10 and having to exchange.</p> <p style="text-align: center;">$41 - 26 =$</p> 	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p> 	<p>Formal column method using 'Steps to Success'. Children must understand what has happened when they have crossed out digits.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <p style="text-align: center;">- Subtraction -</p> <table style="border-collapse: collapse; margin: 0 auto;"> <tr> <td style="padding: 2px 5px;">H</td><td style="padding: 2px 5px;">T</td><td style="padding: 2px 5px;">O</td> <td style="padding: 2px 5px;">T</td><td style="padding: 2px 5px;">O</td> <td style="padding: 2px 5px;">H</td><td style="padding: 2px 5px;">T</td><td style="padding: 2px 5px;">O</td> </tr> <tr> <td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">6</td><td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">=</td> <td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">3</td><td style="padding: 2px 5px;">5</td> </tr> </table> </div> <div> <p>Start in your ones. If you can't do it, exchange 10 or 100 across.</p> <p>Remember to keep your exchanges small and tidy so you don't get confused!</p> </div> </div>	H	T	O	T	O	H	T	O	1	6	2	2	7	=	1	3	5
H	T	O	T	O	H	T	O													
1	6	2	2	7	=	1	3	5												

Years 4-6 – Subtraction

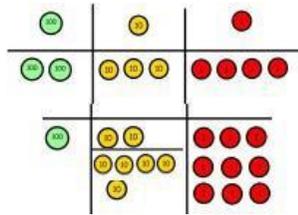
Objective & Strategy	Concrete	Pictorial	Abstract
<p style="text-align: center;"><u>Year</u> <u>4</u></p> <p>Subtracting tens and ones – up to 4 digits.</p> <p>(introduce decimal subtraction through context of money)</p>	<p>Model process of exchange using numicon, base ten and then move to place value counters.</p> <p style="text-align: center;">$234 - 179 =$</p>	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p>	<p>Formal column method. Children must understand what has happened when they have crossed out digits.</p>
<p style="text-align: center;"><u>Year</u> <u>5</u></p> <p>Subtract with at least 4 digits, including money and measures.</p> <p>(subtract with decimal values, including mixtures of integers and decimals and aligning the decimal)</p>	<p>Model process of exchange using numicon, base ten and then move to place value counters.</p> <p style="text-align: center;">$234 - 179 =$</p>	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p>	<p>Formal column method. Children must understand what has happened when they have crossed out digits. Use zeros for place holders.</p>

Year 6

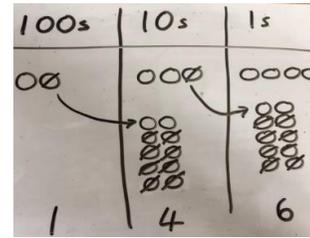
Subtract with increasingly large, more complex, numbers and decimal values.

Model process of exchange using numicon, base ten and then move to place value counters.

$$234 - 179 =$$



Represent the place value counters pictorially; remembering to show what has been exchanged.



Increasingly large and more complex numbers.

$$\begin{array}{r} \cancel{7} \cancel{8} \cancel{10}, 699 \\ - 89,949 \\ \hline 60,750 \end{array}$$

$$\begin{array}{r} \cancel{7} \cancel{10} 5 \cdot \cancel{4} 19 \text{ kg} \\ - 36 \cdot 080 \text{ kg} \\ \hline 69 \cdot 339 \text{ kg} \end{array}$$

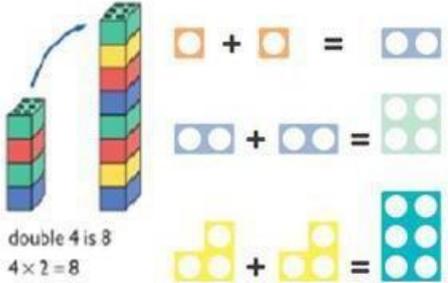
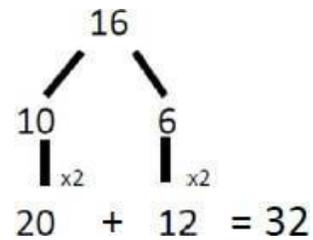
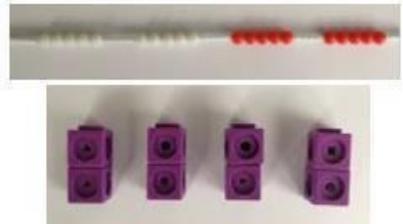
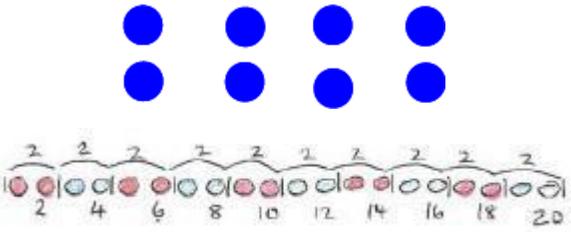
Progression in Calculations – Multiplication

Nursery / Pre-School and Reception

By the end of Reception, children are expected to understand the concept of doubling and to be able to double a number up to 10. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out 'doubling' by physically add two equal groups together to find out the 'doubles' answer.

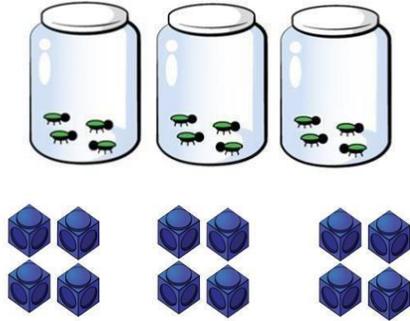


EYFS - Year 1 – Multiplication

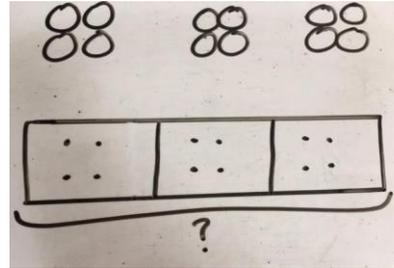
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling numbers.</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.</p>  <p>The image shows a stack of 4 cubes being doubled to a stack of 8 cubes. Below, Numicon blocks illustrate: 2 orange blocks + 2 orange blocks = 4 blue blocks; 2 blue blocks + 2 blue blocks = 4 green blocks; 2 yellow blocks + 2 yellow blocks = 4 teal blocks.</p>	<p>Draw pictures to show how to double numbers.</p> <p style="text-align: center;">Double 4 is 8</p>  <p>Two groups of 4 purple squares are shown, representing 4 + 4 = 8.</p>	<p>Partition a number and then double each part before recombining it back together.</p>  <p>A tree diagram shows 16 branching into 10 and 6. Below, 10 is multiplied by 2 to get 20, and 6 is multiplied by 2 to get 12. Finally, 20 + 12 = 32.</p>
<p>Counting in multiples.</p>	<p>Count the group as children are skip counting, children may use their fingers to help.</p>  <p>A bead string with red beads is shown, along with four purple Numicon blocks representing the sequence 2, 4, 6, 8.</p>	<p>Children make representations to show counting in multiples.</p>  <p>Two rows of 4 blue dots each represent 2 groups of 2. Below, a number line from 0 to 20 shows skip counting by 2s: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Repeated grouping/repeated addition.

$3 \times 4 =$
 $4 + 4 + 4 =$
There are 3 equal groups, with 4 in each group.



Children to represent the practical resources in a picture and use a bar model.

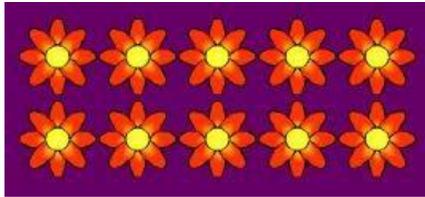


$$3 \times 4 = 12$$

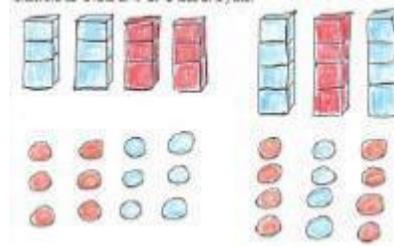
$$4 + 4 + 4 = 12$$

Understanding arrays.

Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2s.



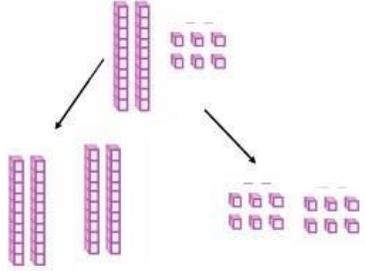
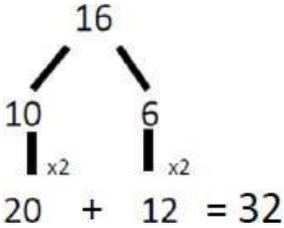
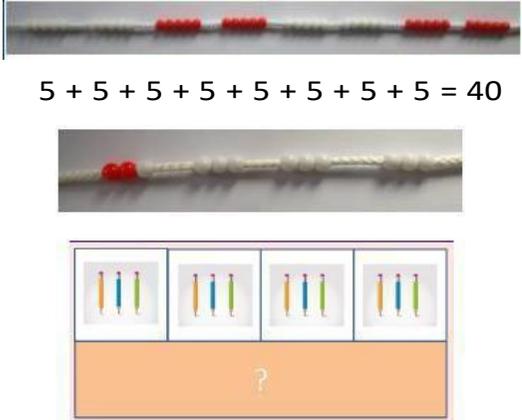
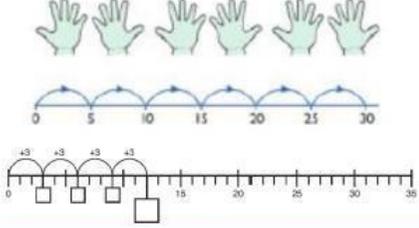
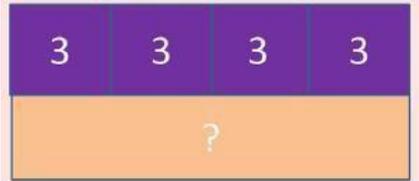
Draw representations of arrays to demonstrate understanding.



$$3 \times 2 = 6$$

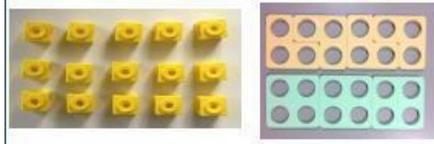
$$2 \times 5 = 10$$

Year 2 – Multiplication

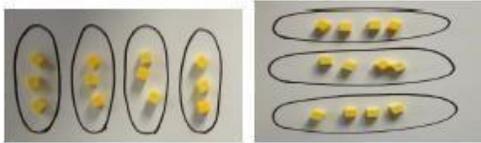
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling numbers.</p>	<p>Model doubling using dienes and place value counters.</p> <p style="text-align: center;">Doubling 26</p> 	<p>Draw pictures and representations to demonstrate how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p> <div style="text-align: center;">  </div>
<p>Counting in multiples of 2, 5 and 10 from 0.</p> <p>(repeated addition)</p>	<p>Count the groups as children are skip counting, children may use their fingers to help. Progress onto bar models.</p>  <p style="text-align: center;">$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$</p>	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>  <div style="text-align: center;">  </div>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> <p style="text-align: center;">$4 \times 3 = \underline{\quad}$</p>

Multiplication is commutative.

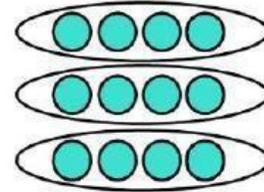
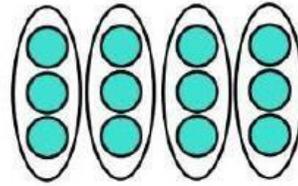
Create arrays using counters, cubes and numicon.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not change the answer.



Use representations of arrays to show different calculations and explore commutativity.



$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

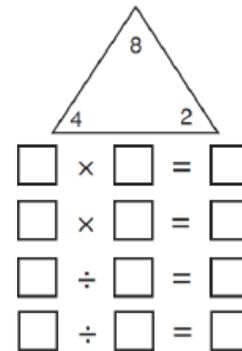
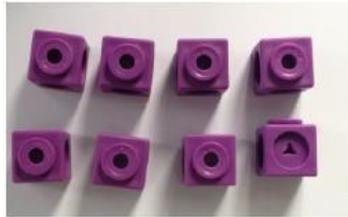
$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Using the inverse.

(this should be taught alongside division, so pupils learn how the two operations work alongside each other)



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

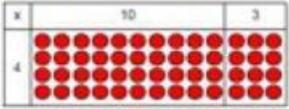
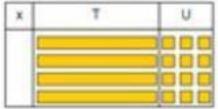
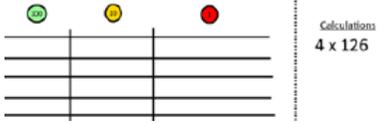
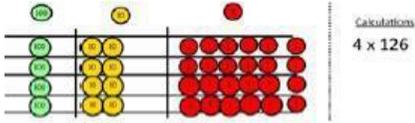
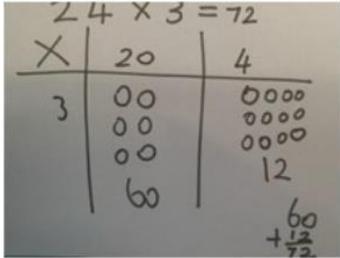
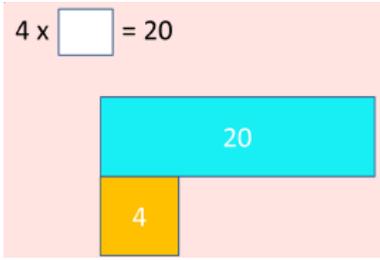
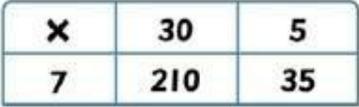
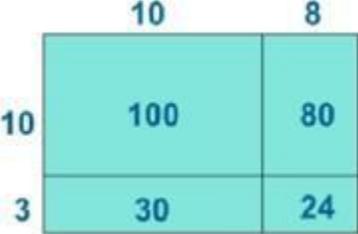
$$8 = 4 \times 2$$

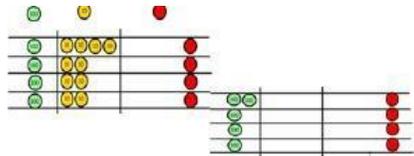
$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

Show all 8 related fact family sentences.

Year 3 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
<p>The grid method.</p>	<p>Show the links with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move onto place value counters to show how we are finding groups of a number. We are multiplying by 4, so we need 4 rows...</p>  <p>Calculations 4×126</p> <p>Fill each row with 126...</p>  <p>Calculations 4×126</p> <p>Add up each column, starting with the ones making any exchanges needed.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colour to show different amounts or just use the circles in the different columns to show their thinking.</p>  <p>Bar models are used to explore missing numbers.</p> 	<p>Begin with multiplying by one digit numbers and showing the clear addition alongside.</p>  <p>$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number, showing the different rows within the grid method.</p> 



Then you have your answer.

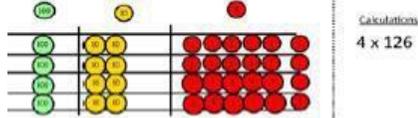
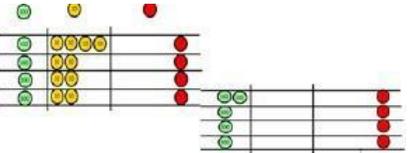
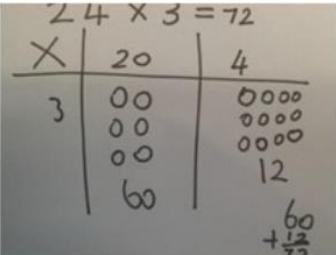
Rapid Recall
(multiplication and
division)

- Multiplication and division facts for 2, 5, 10, 3, 4 and 8 times tables.

Strategies

- 'Double-double'/'half-half' links within the listed times tables.
- Associativity (pushing numbers around)
- Using what I already know

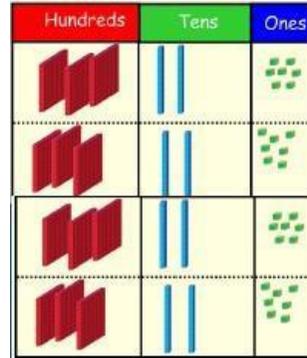
Year 4 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract								
<p>The grid method (recap from Year 3 for 2 digit x 1 digit).</p> <p>Children progress to multiplying 3 digit numbers by 1 digit (Year 4 expectation).</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p>  <p>Fill each row with 126.</p>  <p>Add up each column, starting with the ones making any exchanges needed.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colour to show different amounts or just use the circles in the different columns to show their thinking.</p> 	<p>Multiply 3 digit by 1 digit numbers using the grid method.</p> <table border="1" data-bbox="1648 580 2107 692"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> <p>$1200 + 80 + 28 = 1,308$</p>	x	300	20	7	4	1200	80	28
x	300	20	7								
4	1200	80	28								

Column Multiplication.

Children can continue to be supported by place value counters at this stage of multiplication. This is initially done where there is no regrouping.

$321 \times 2 = 642$



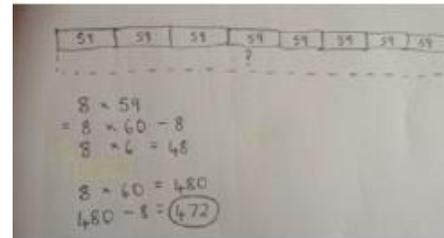
It is important at this stage that they always multiply the ones column first.

The corresponding long multiplication is modelled alongside this method.

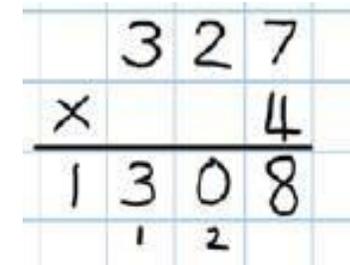
The grid method may be used to show how this relates to a formal written method (see abstract column).

x	300	20	7
4	1200	80	28

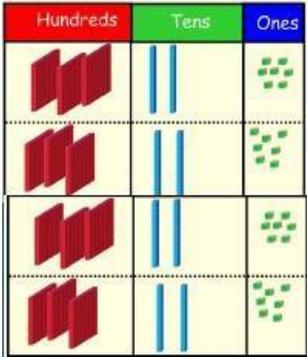
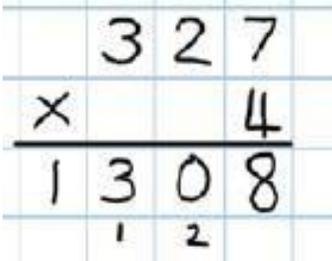
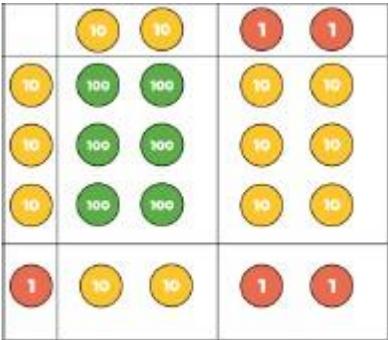
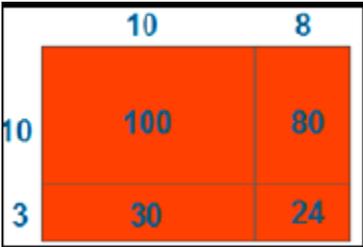
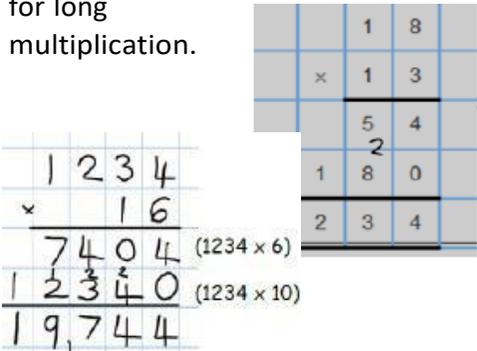
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



The grid method can then be progressed onto the compact method.



Year 5 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract								
<p>Column Multiplication (3 and 4 digits x 1 digit).</p>	<p>Children can continue to be supported by place value counters at this stage of multiplication. This is initially done where there is no regrouping.</p> 	<p>The grid method may be used to show how this relates to a formal written method (see abstract column).</p> <table border="1" data-bbox="1032 563 1565 692"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table>	x	300	20	7	4	1200	80	28	<p>The grid method can then be progressed onto the compact method.</p> 
x	300	20	7								
4	1200	80	28								
<p>Column Multiplication – Long multiplication.</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside. (22 x 31)</p> 	 <p>Continue to use bar modelling to support problem solving.</p>	<p>Progress to using the column method for long multiplication.</p> 								

Rapid Recall

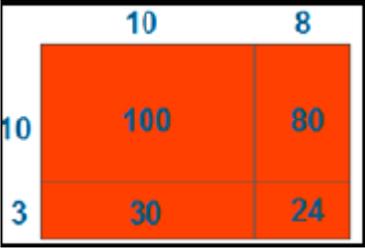
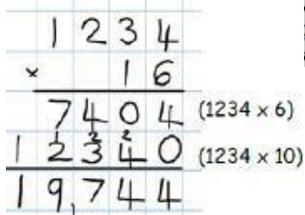
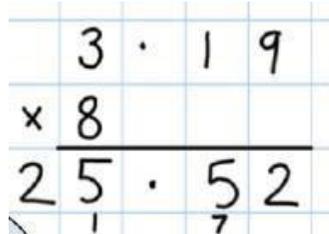
(multiplication and division)

- Square numbers to 144
- Establish whether a number is prime
- Recall all prime numbers up to 19

Strategies

- X by 9
- $X \div$ by 10/100/1000
– including decimals
- Use what you know to...
- $x \div$ by 5/50/25
- x by $\frac{1}{2}$
- Use factor pairs – 24 ~~x~~ 16

Year 6 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Multiplication – Long multiplication.</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>	<div style="text-align: center;">  </div> <p>Continue to use bar modelling to support problem solving.</p>	<p>Progress to using the column method for long multiplication.</p> <div style="text-align: center;">  </div>
<p>Multiplying decimals up to 2 decimal places by a single digit.</p>			<p>Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and answer.</p> <div style="text-align: center;">  </div>

When appropriate, children can use their place value knowledge to make the number being multiplied 10, 100 or 1000 times bigger and then multiply and make the answer 10, 100 or 1000 times smaller.

$$\begin{array}{r} 319^{(x100)} \\ \times \quad 8 \\ \hline 2552^{(+100)} \end{array} = 25.52$$

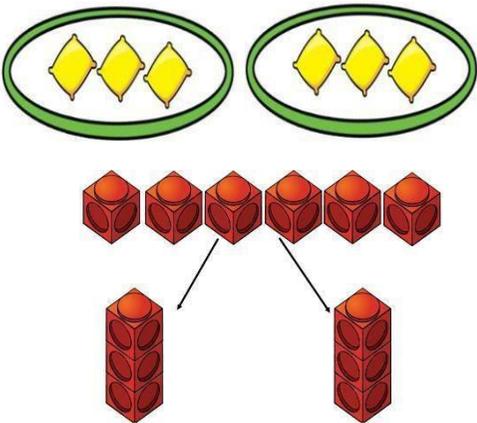
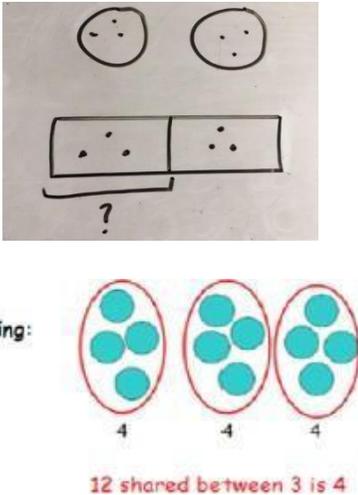
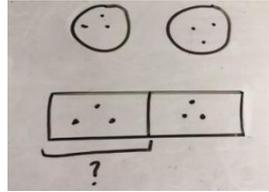
Progression in Calculations – Division

Nursery / Pre-School and Reception

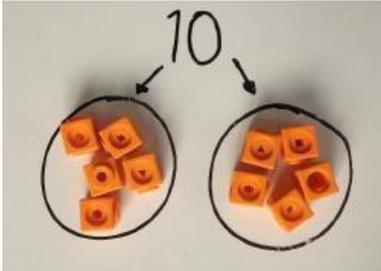
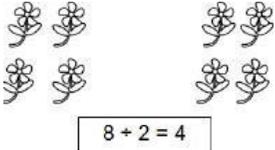
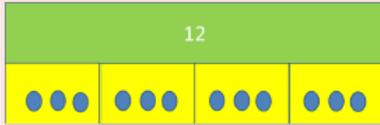
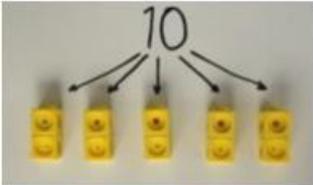
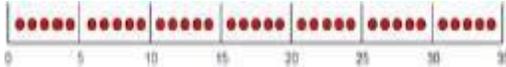
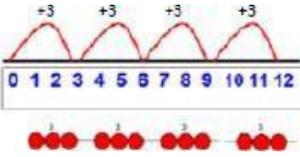
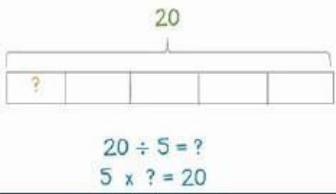
By the end of Reception, children are expected to understand the concept of halving and sharing. Before this can be introduced, children need to have a secure knowledge of counting backwards, number facts and subtraction in order to halve and share. Children are then introduced to the concept of halving and sharing through practical games and activities. They act out 'halving and sharing' through activities such as sharing food for their Teddy Bear's Picnic, sharing resources equally to play a game. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.



EYFS - Year 1 – Division

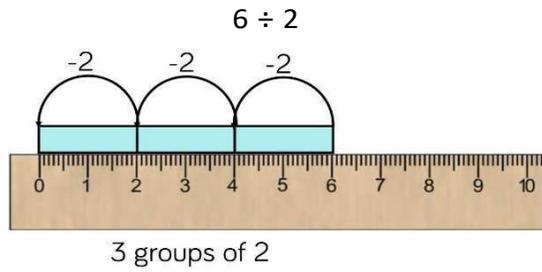
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	<p>Sharing using a range of objects: $6 \div 2 =$</p> 	<p>Use pictures or shapes to share quantities:</p>  <p>Sharing:</p> <p>12 shared between 3 is 4</p>	<p>Children continue with pictorial method until fully secure. Children should also be encouraged to use their 2 times tables facts.</p>  <p>To progress further, children can then be moved onto:</p> <p>'6 shared between 2 is 3'</p>

Year 2 – Division

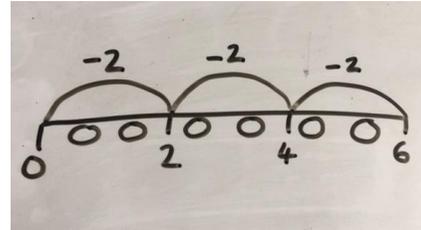
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	<p>I have 10 cubes, can you share them into 2 equal groups?</p> 	<p>Children use pictures or shapes to share quantities:</p>  <p>Children use bar modelling to show and support understanding:</p> $12 \div 4 = 3$ 	$12 \div 3 = 4$
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Use number lines for grouping:</p>  <p>Use bar model to support with division:</p>  $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p>

Division using repeated subtraction

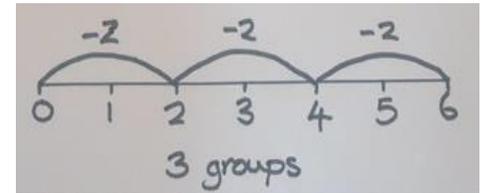
Repeated subtraction using Cuisenaire rods above a ruler:



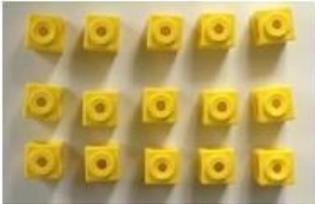
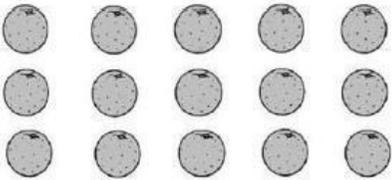
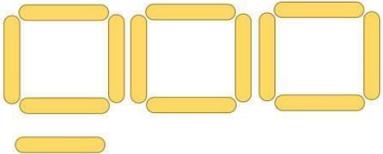
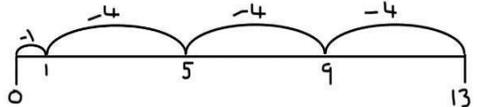
Children to represent repeated subtraction pictorially:



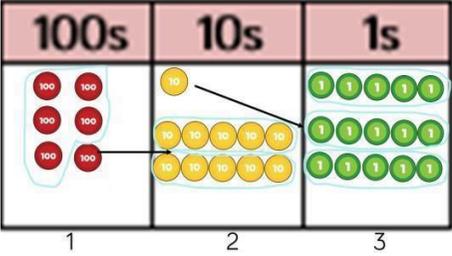
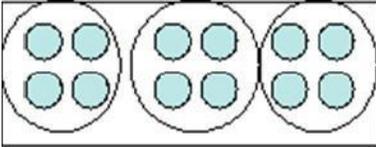
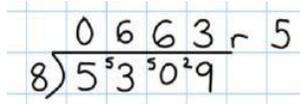
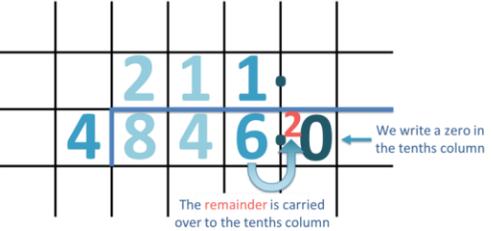
Abstract number line to represent the equal groups that have been subtracted:



Year 3 – Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division with arrays</p>	<p>Link division to multiplication by creating an array and thinking about the number sentences that can be created:</p>  <p style="text-align: center;"> $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$ </p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences:</p>  <p style="text-align: center;"> $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$ </p>	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences:</p> <p style="text-align: center;"> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$ </p>
<p>Division with remainders</p>	<p>This can be done with lollipop sticks or Cuisenaire rods:</p> <p style="text-align: center;">$13 \div 4$</p>  <p>Use of lollipop sticks to form wholes-squares are made because we are dividing by 4.</p> <p>There are 3 whole squares, with 1 left over.</p>	<p>Children to represent the lollipop sticks pictorially:</p>  <p>There are 3 whole squares, with 1 left over.</p>	<p style="text-align: center;">$13 \div 4 = 3$ remainder 1</p> <p>Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line:</p>  <p style="text-align: center;">‘3 groups of 4, with 1 left over’</p>

Year 4-6 – Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Short division with a remainder</p> <p><u>Year 4</u> Up to 3 digits by 1 digit</p> <p><u>Year 5</u> Up to 4 digits by a 1 digit with remainders</p> <p><u>Year 6</u> Up to 4 digits by a 1 digit and then progress to long division (next objective)</p>	<p>Short division using place value counters to group:</p> <p style="text-align: center;">$615 \div 5$</p>  <ol style="list-style-type: none"> 1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones? 	<p>Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups:</p>  <p>However, children should be encouraged to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainders:</p> $\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \underline{8} \\ 7 \\ \underline{8} \\ 2 \\ \underline{2} \\ 0 \end{array}$ <p>Move onto divisions with a remainder:</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{4} \\ 3 \\ \underline{3} \\ 2 \\ \underline{2} \\ 0 \end{array}$  <p><u>Year 5/6</u></p> <p>Children can then progress onto expressing the remainder as fractions (e.g. 5/8) and decimals (e.g. 663.625).</p> <p style="text-align: center;">$846 \div 4$</p> 

Year 6 – Division

Long division with remainder

Begin by modelling method with a 1-digit divisor.

Long Division

Divide :	$\begin{array}{r} 2 \rightarrow \\ 3 \overline{)74} \\ \underline{0} \\ 74 \end{array}$ <p>Dividing 7 tens by 3, we get 2 tens, and some extra.</p>
Multiply :	$\begin{array}{r} 2 \\ 3 \overline{)74} \\ \underline{6} \\ 6 \end{array}$ <p>$6 \rightarrow 3 \times 2 \text{ tens} = 60 \text{ tens.}$</p>
Subtract :	$\begin{array}{r} 2 \\ 3 \overline{)74} \\ \underline{-6} \\ 1 \end{array}$ <p>Subtracting 6 tens from 7 tens</p>
Bring down :	$\begin{array}{r} 2 \\ 3 \overline{)74} \\ \underline{-6} \\ 14 \end{array}$ <p>$14 \rightarrow 1 \text{ ten } 4 \text{ ones} = 14 \text{ ones}$</p>
Repeat or find the Remainder :	$\begin{array}{r} 24 \\ 3 \overline{)74} \\ \underline{-6} \\ 14 \\ \underline{-12} \\ 2 \end{array}$ <p>Dividing 14 ones by 3, we get 4 ones and some extra. $3 \times 4 \text{ ones} = 12 \text{ ones.}$ Remainder</p>
Check :	<p>Check your answer: Dividend = Divisor \times Quotient + Remainder</p>

Divide- the number inside the house with the number outside of the house. Put the answer on top.

Multiply –the number outside of the house by the number on top of the house. Put this answer below the number inside the house.

Subtract- the number inside the house from the number below the inside number.

Bring down- the next number in the dividend.

Repeat- all the steps repeated as many times as needed until you get down to 0.
 If there is a leftover this is your remainder.

When moving onto using long division with a 2-digit divisor, children can write out multiples first:

2	2	6	7	1	0		

22, 44, 66, 88, 110 etc

Long division with
decimal remainders

$$\begin{array}{r} 25.2 \\ 5 \overline{) 126.0} \\ \underline{-10} \\ 26 \\ \underline{-25} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

When there is a remainder which you need to write as a decimal, bring down the 0 in the from then tenths column, and repeat the process as before.

Signed on behalf of the governing body:

Date:

It will be reviewed in Summer Term 2025